THE NON SELF SIMILAR SCALING OF VORTICITY IN A SHEAR LAYER

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Abstract: Time series measurements of the spanwise vorticity fluctuations in a large single stream shear layer have been acquired. The magnitude of the fluctuations, as measured by the standard deviation, was normalized using dissipation variables and compared at two streamwise locations. These data indicate that the vorticity fluctuations do not follow a self similar form. Rather, it was found that the values increase in magnitude with downstream distance. An explanation of this phenomenon is provided.

Keywords: self similar, vorticity, shear layer.

1 INTRODUCTION AND EXPERIMENT

The principal of self similarity as a representation of moving equilibrium was introduced by Townsend (1956). Free shear layers provide an excellent example of this equilibrium, and they form one class of the canonical turbulent flow fields. A dimensionless variable that is written as a function of a dimensionless transverse coordinate is called self similar if the function does not change with downstream position. Recent vorticity measurements acquired in a large single stream shear layer will be used to support the following hypothesis. *The vorticity fluctuations in a shear layer do not scale in a self similar manner. Instead, these values increase with the logarithm of the streamwise location when scaled using the dissipation time scales*.

In previous research efforts, vorticity fluctuations in shear layers have been scaled with either the integral time scale (θ/U_o), or the dissipation time scale $\sqrt{\nu/\varepsilon}$. Ballint and Wallace (1988), and Haw et al. (1989) used the integral time scales, and found that the normalized vorticity fluctuations increased dramatically with downstream distance. Since $|\vec{\omega}|^2 = \varepsilon / \nu$ in homogeneous turbulence, the dissipation time scales are often chosen to represent the vorticity fluctuations, see, e.g., Rogers and Moser (1994), and Loucks (1998). These studies have found that the vorticity roughly scales in a self similar manner.

The present experiments have shown that scaling vorticity with dissipation scales yields values which increase slowly with downstream distance (note that the dimensional values decrease with x). This increase has not previously been documented because of the relatively large streamwise development length required to observe an increase above the experimental uncertainty. A large scale single stream shear layer facility has been constructed at the Turbulent Shear Flows Laboratory at Michigan State University, see Figure 1. The Reynolds number of the boundary layer at separation is Re_θ=4650, based on the momentum thickness $\theta(x=0)=\theta_o$=9.6mm, and the constant free stream velocity U_o=7.1m/s. The shear layer test section is 9.7m in length from the separation point to the tunnel exit.

Spanwise vorticity measurements have been acquired using the compact four sensor hot-wire probe (Fig. 2) developed by Foss and coworkers, see, e.g., Haw et al. (1989). The two parallel wires record the magnitude of velocity with a small ($\delta y \sim 1.4$mm) spatial separation. The sensors configured as an X-array are used to recover the v component of velocity. From these three signals, a micro circulation domain is constructed with the flow direction calculated using a convected distance from the local (in time) velocity magnitude (Fig. 2). The vorticity probe was traversed across the shear layer at two streamwise locations: x/θ_o= 384 and 675. Time series data were acquired at 40kHz for 25 seconds.

2 RESULTS AND CONCLUSIONS

It is useful to consider the budget and scaling of the turbulent kinetic energy (TKE). The experiment described above provides information to calculate each of the terms except the pressure-velocity correlation terms, which were determined from the balance of the measured terms of the TKE equation (see, e.g., Wygnanski and Fiedler (1970) or Morris(2002)). These data are shown in Figure 3 scaled by θ / U_o^3. The dissipation term was calculated from the isotropic relation: $\varepsilon \approx 30 \nu \overline{u'^2} / \lambda_u^2$. The transverse coordinate of the shear layer is represented by $\eta = y/\theta$, where $\theta(x)$ is the local momentum thickness of the shear layer. The collapse of the data indicates that all of the terms scale in a self similar manner as expected. The argument can then be made that the dissipation time scale can be related to the integral quantities of the shear layer by $\sqrt{\nu / \varepsilon} \sim \sqrt{\nu \theta / U_o^3}$.

The profiles of the standard deviation of the vorticity time series data are shown in Figure 4. An increase in the peak vorticity of 15% can be observed between the two streamwise locations. It is argued that this increase is a general characteristic of free shear layer flows. Note that the vorticity probe resolution ($\delta y/\eta_K$) at the up and downstream locations was found to be 7.8 and 7.0, respectively, where η_K represents the Kolmogorov length scale.

The physical mechanism by which the vorticity fluctuations increase can be considered in terms of the auto spectral density of the vorticity time series. These data are shown in Figure 5 for the spatial locations corresponding to the peak vorticity (roughly $\eta=0$). The spectra are normalized by the dissipation scales. Also included in Figure 5 is the vorticity spectra calculated by fitting the one dimensional velocity spectra $E_{11}(k_1)$, and using isotropic relationships to arrive at $\phi_\omega(k_1)$.

Several distinctive features can be observed from the vorticity spectra. First, it is noted that the distributions at the two spatial locations agree well at the dissipation length scales, which indicates that the probe resolution is adequate to resolve the small scale vorticity motions. Second, a region can be observed $[2 \times 10^{-3} < k_1 \eta_K < 0.1]$ where the vorticity spectra follows a k^{-1} power distribution. This is in contrast to isotropic turbulence where there is no log-linear region in the one dimensional spectra.

An additional feature that can be observed in Figure 5 is the larger spectral values in the low wave number range at the downstream location. Observations made in the low wave number region (i.e., large scale motions) are often more instructive when viewing the inverse Fourier transform of the spectra, that is, the auto correlation function: $R_{\omega\omega}$. These data are shown in Figure 6 where time is normalized by the integral time scale (θ/U_o). These data show that the vorticity correlation magnitudes at different streamwise locations are identical at large time delay when time is scaled by (θ/U_o). Specifically the first zero of the autocorrelation is found at a dimensionless time of $t\theta/U_o \approx 2.5$, which was also found by Bruns et al. (1991).

The data have shown that the vorticity spectra at $x/\theta(0)=384$ and 675, scaled by dissipation variables, correspond at the high wave numbers and deviate at the low wave numbers (Figure 5). The latter observation is quantitatively supported by the correspondence of the vorticity autocorrelation functions when scales with U_o/θ (Figure 6). The increase in the vorticity fluctuations with downstream distance can now be explained by representing the variance of the vorticity fluctuations as the integral of the spectral density function:

$$(\tilde{\omega}_z^*)^2 = \int_{\log(k_1\theta)}^{\log(k_1\eta_K)} [k_1\eta_K \cdot \phi^*_{\omega_z}(k_1)]\, d(\log(k_1\eta_K)) \qquad (1)$$

where the superscript * represents a quantity made dimensionless by dissipation scales. The limits of integration are justified given that the vorticity spectra does not contain significant values for wave numbers smaller than the integral scales or larger than the Kolmogorov scales. If it were assumed that the integrand (in square brackets) is roughly equal to a constant (see Figure 5) over this range of integration, then equation (1) implies that the vorticity variance will increase as the difference between the large scale (θ) and small scale (η_K) motions increases. In a single stream shear layer this ratio increases with streamwise position (at a rate of $\theta/\eta_K \sim x^{3/4}$).

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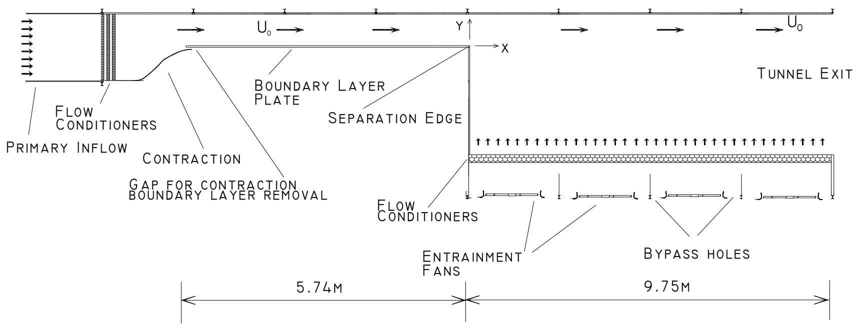

*Figure 1.*Schematic of the Single Stream Shear Layer Facility

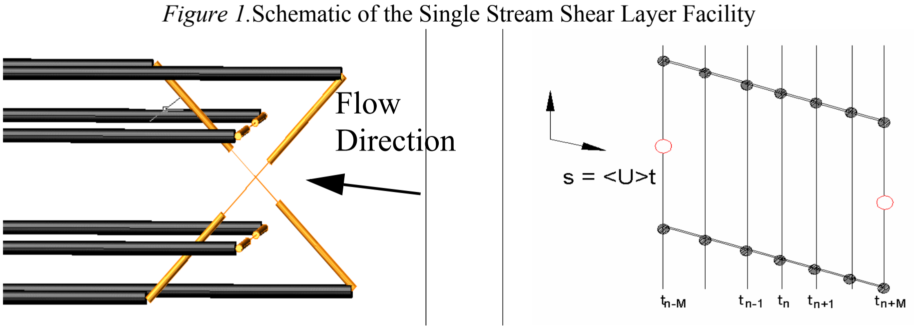

*Figure 2.*Schematic of vorticity probe and micro circulation domain

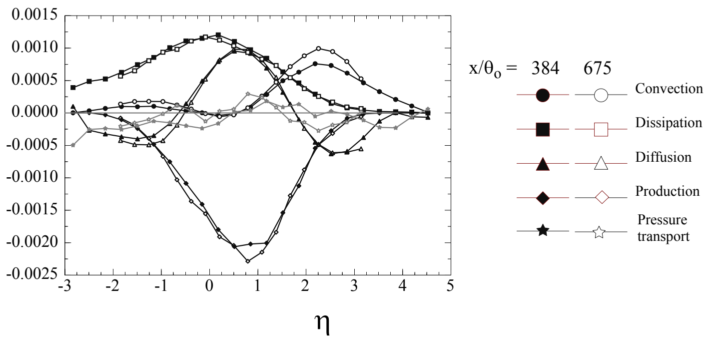

*Figure 3.*Measured terms of the turbulent kinetic energy budget

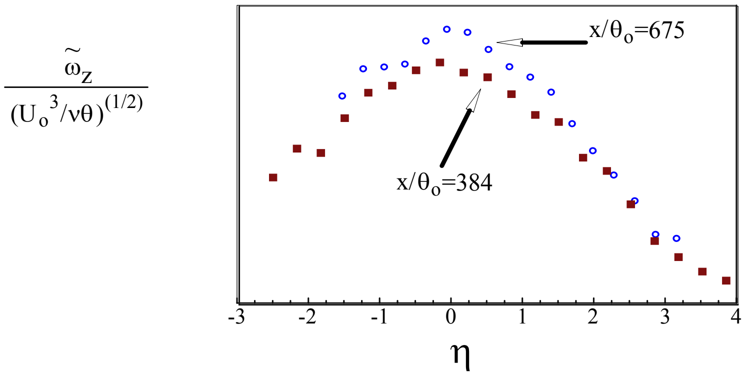

Figure 4. Profiles of the dimensionless vorticity fluctuations

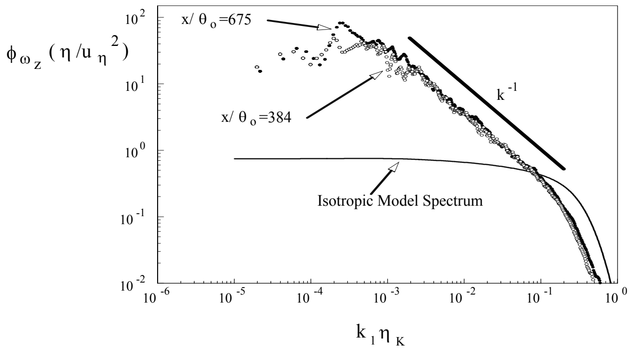

Figure 5. Autospectral density of the vorticity at the locations of peak fluctuation level normalized by dissipation scales. Note that η_K represents the Kolmogorov length scale.

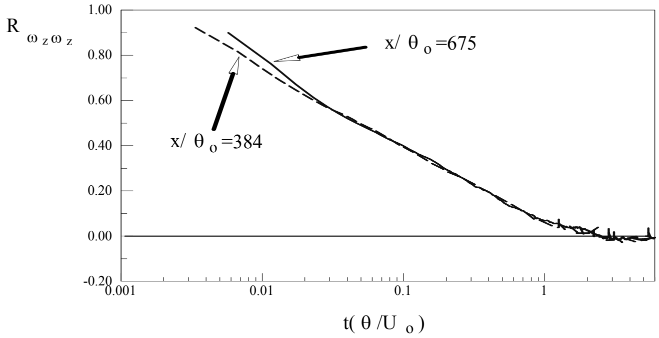

Figure 6. Autocorrelation function of the vorticity